SG6: High Performance Computing

Distributed algorithms on multi-dimensional topologies

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Distributed algorithms on multi-dimensional topologies

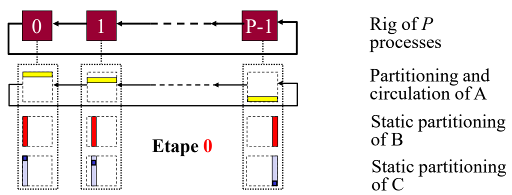
Dense matrix product on a 1D ring
- **Parallelisation scheme**
- **Performance model**

Dense matrix product on a 2D torus
Hyper-quicksort on a kD hypercube

Parallelisation scheme (C = A×B)

Data partitioning and circulation on a 1D ring:

- A: partitioned into line blocks
- B, C: partitioned into column blocks
- **Circulation of A slices**
- Static B and C slices

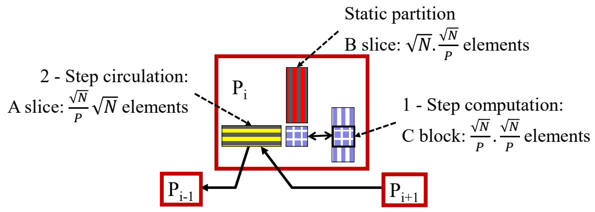


Notations:
- Matrixes with: $N = n \times n$ elements
- P processes
- Matrixes slices of: $\sqrt{N}.\sqrt{N}/P = N/P$ elements

Parallelisation scheme (C = A×B)

At each of the P stages:

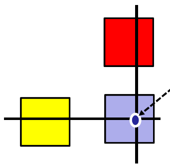
- All processes are working (at any moment)
- In each process (and at each stage):
  1 - calculation of a small square block of C
  2 - circulation of a « horizontal » slice of A



Dense matrix product on a 1D ring

Performance model

Sequential computation of C matrix:



A, B and C: $\sqrt{N} \times \sqrt{N}$ elements

C_{ij} computation:
$\sqrt{N}$ multiplications
and ($\sqrt{N}$-1) additions

$$T^{seq}_{Cij\ comput} = \sqrt{N}.t_{mult} + (\sqrt{N} - 1).t_{add}$$

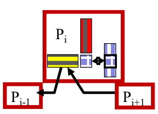
$$T^{seq}_{C\ comput} = (\sqrt{N}.\sqrt{N}).(\sqrt{N}.t_{mult} + (\sqrt{N} - 1).t_{add})$$

$$\Rightarrow \boldsymbol{T^{seq}_{C\ comput} \approx N.(2.\sqrt{N}.t_{flop})}$$

Dense matrix product on a 1D ring

Performance model

Distributed computation of C matrix:



During one step:

Computation of a C block of $\frac{\sqrt{N}}{P}.\frac{\sqrt{N}}{P}$ elements

- Time on one process: $t^{1-process}_{1-comput} \approx \frac{\sqrt{N}}{P}.\frac{\sqrt{N}}{P}.(2.\sqrt{N}.t_{flop})$
- Time on the *P* processes:

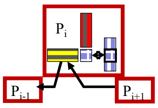
hypothesis {
perfect load balancing,
identical computing resources
perfect synchronization

$$\Rightarrow \boldsymbol{t^{P-process}_{1-comput} = t^{1-process}_{1-comput}}$$

$$\Rightarrow t^{P-process}_{1-comput} \approx \frac{N}{P^2}.(2.\sqrt{N}.t_{flop})$$

Performance model

Distributed computation of C matrix:



During one step:

Circulation of A slices of $\sqrt{N}.\frac{\sqrt{N}}{P}$ elements

- Time of one communication: $P_{i-1} \leftarrow P_i$

$$t_{1-comm} = t_s + (\sqrt{N}.\frac{\sqrt{N}}{P}).t_w \quad \begin{cases} t_s \text{ : setup time or latency time} \\ t_w = sizeof(1\ element)/B_w \end{cases}$$

- Time of all the communications accross the P processes (during 1 step)

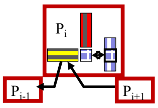
hypothesis
- *All communications are executed in parallel,*
- → *each process can send and receive in parallel…*
  - *…depends on the hardware & on the MPI comm routine!*
- → *all communications cross the same « network »…*
  - *… depends on the deployment!*

⇨ $\boldsymbol{t_{1-circulation}^{P-processes} = t_{1-comm} \approx \frac{N}{P}.t_w}$

Dense matrix product on a 1D ring

Performance model

Distributed computation of C matrix:



During one step:

$$t_{1-step}^{P-process} = t_{1-comput}^{P-process} + t_{1-circulation}^{P-processes}$$

$$t_{1-step}^{P-process} \approx \frac{N}{P^2}.(2.\sqrt{N}.t_{flop}) + \frac{N}{P}.t_w$$

During the *P* steps of C distributed computation:

$$T_{C\ comput}^{distributed} = P.\,t_{1-step}^{P-process}$$

⇨ $\boldsymbol{T_{C\ comput}^{distributed} \approx \frac{N}{P}.(2.\sqrt{N}.t_{flop}) + N.t_w}$ **with: *P > 1***

hypothesis
- *All computations identical & executed in parallel*
- *All communications identical & executed in parallel*

Dense matrix product on a 1D ring

Performance model

Distributed computation of C matrix:

Execution times:

$$T_{C\ comput}^{seq} \approx N.(2.\sqrt{N}.t_{flop})$$

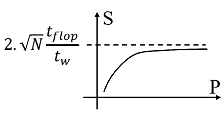
$$T_{C\ comput}^{distributed} \approx \frac{N}{P}.(2.\sqrt{N}.t_{flop}) + N.t_w \qquad \text{with: } P > 1$$

Speedup:

$$S(N,P) = \frac{T_{C\ comput}^{seq}}{T_{C\ comput}^{distributed}} = \frac{N.(2.\sqrt{N}.t_{flop})}{\frac{N}{P}.(2.\sqrt{N}.t_{flop}) + N.t_w}$$

⇨ $\boldsymbol{S(N,P) = P.\frac{1}{1 + \frac{P}{2.\sqrt{N}} \times \frac{t_w}{t_{flop}}}}$

With N = N_0

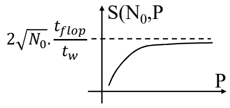


Performance model

Distributed computation of C matrix:

$$S(N,P) = P \cdot \frac{1}{1 + \frac{P}{2.\sqrt{N}} \times \frac{t_w}{t_{flop}}}$$

- **When data size is fixed:** $N = N_0$



- **When $P = P_0$ is fixed**, and data size increases:

$\lim_{N\to\infty} (S(N, P_0)) = P_0$ Speedup increases and tends to P_0 (ideal S)

In fact: $T_{C\ comput}^{distributed} \approx \underbrace{\frac{N}{P_0}.(2.\sqrt{N}.t_{flop})}_{O(N^{3/2})} + \underbrace{N.t_w}_{O(N)}$

When increasing the pb size, on a fixed number of nodes:
Computations increase faster than communications
→ Speedup will increase! …. Nice problem!

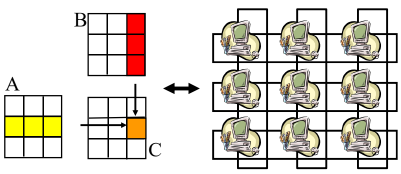
Distributed algorithms on multi-dimensional topologies

Dense matrix product on a 1D ring
Dense matrix product on a 2D torus
- **Parallelisation scheme**
- **Performance model**

Hyper-quicksort on a kD hypercube

Parallelisation scheme (C = A×B)

A, B, C : n × n = N elements C = A × B $c_{ij} = \sum_{k=1}^{n} (a_{ik} b_{kj})$



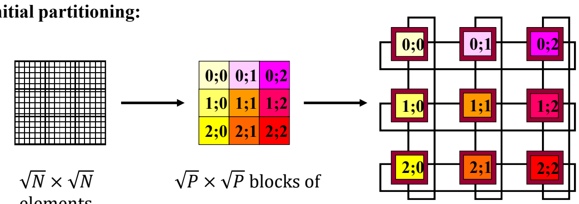
$\sqrt{N} \times \sqrt{N}$ elements $\sqrt{P} \times \sqrt{P}$ processes

→ Which data partitioning & circulation ?

Dense matrix product on a 2D torus

Parallelisation scheme (C = A×B)

Initial partitioning:



$\sqrt{N} \times \sqrt{N}$ elements

$\sqrt{P} \times \sqrt{P}$ blocks of $\frac{\sqrt{N}}{\sqrt{P}} \times \frac{\sqrt{N}}{\sqrt{P}}$ elements

$\sqrt{P} \times \sqrt{P}$ processes

Only one partitioning for A, B and C

→ Result matrix C can be reused as a left or right operand

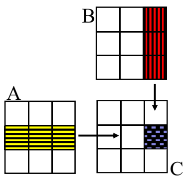
Dense matrix product on a 2D torus

Parallelisation scheme (C = A×B)

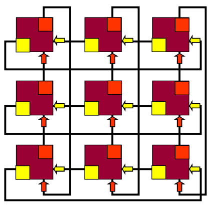
Data circulation principle:

A blocks: require a **circulation on the row** of processes ⇦

B blocks: require a **circulation on the colums** of processes ⇧



$$C_{I,J} = \sum_{K=1}^{\sqrt{P}} (A_{I,K} \times B_{K,J})$$



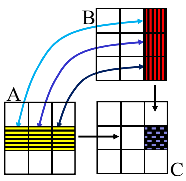
→ Each process will receive all data it needs to compute its C block

Dense matrix product on a 2D torus

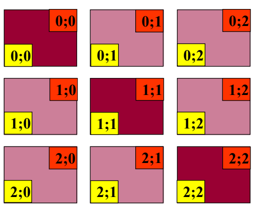
Parallelisation scheme (C = A×B)

But process $P_{I,J}$ requieres simultaneously the blocks $A_{I,K}$ and $B_{K,J}$:

With the initial partitioning only process $P_{I,I}$ can start their computations



$$C_{I,J} = \sum_{K=1}^{\sqrt{P}} (A_{I,K} \times B_{K,J})$$



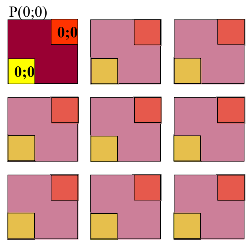
→ We need to improve the initial partitioning

Dense matrix product on a 2D torus

Parallelisation scheme (C = A×B)

Approach:

- We keep the 2D partitioning principle
- We keep the circulation scheme
  (A blocks circulate on process rows, B blocks circulate on process columns)
- But we look for a better initial partitioning:
  - **we install the first blocks required by P(0;0)**
  - **and we propagate the constraints…**



Dense matrix product on a 2D torus

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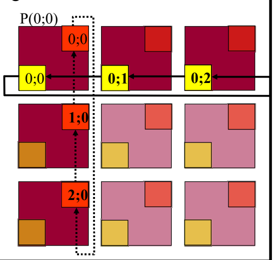
- But we look for a better initial partitioning:
  - we install the first blocks required by P(0;0)
  - **the circulation scheme impose:**
    $A_{0,J}$ on the first line
    $B_{I,0}$ on the first column



Dense matrix product on a 2D torus

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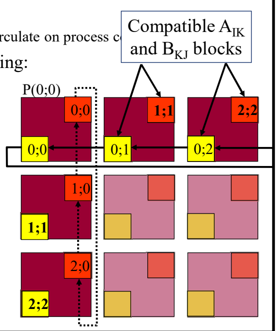
- But we look for a better initial partitioning:
  - we install the first blocks required by P(0;0)
  - the circulation scheme impose
    $A_{0,J}$ on the first line
    $B_{I,0}$ on the first column
  - **we install A & B blocks required on the 1st col and line**

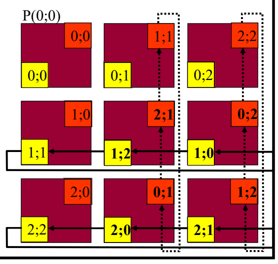


Dense matrix product on a 2D torus

Parallelisation scheme (C = A×B)

Approach:

- We keep the 2D partitioning principle
- We keep the circulation scheme
  (A blocks circulate on process rows, B blocks circulate on process columns)
- But we look for a better initial partitioning:
  - we install the first blocks required by P(0;0)
  - the circulation scheme impose $A_{0,J}$ on the first line $B_{I,0}$ on the first column
  - we install A & B blocks required on the 1st col and line
  - **we install others A & B blocks, guided by circulation order**

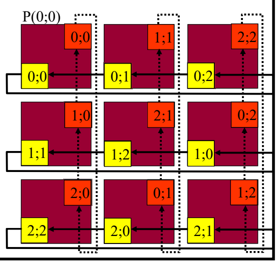


Dense matrix product on a 2D torus

Parallelisation scheme (C = A×B)

Approach:

- We keep the 2D partitioning principle
- We keep the circulation scheme
  (A blocks circulate on process rows, B blocks circulate on process columns)
- **We get an initial partitioning where all A_{IK} and B_{KJ} are compatible**
- **Couples of blocks remain compatible after each circulation step**



Dense matrix product on a 2D torus

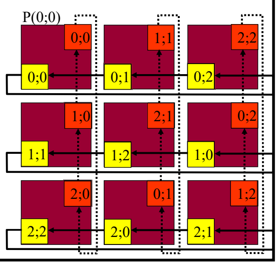
Parallelisation scheme (C = A×B)

Approach:

- We keep the 2D partitioning principle
- We keep the circulation scheme
  (A blocks circulate on process rows, B blocks circulate on process columns)
- We get an initial partitioning where all A_{IK} and B_{KJ} are compatible
- Couples of blocks remain compatible after each circulation step

Finally:

- **Row *I* has been pre-shifted of *I* steps to the left**
- **Column *J* has been pre-shifted of *J* steps to the top**



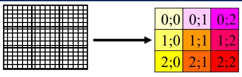
Dense matrix product on a 2D torus

Parallelisation scheme (C = A×B)

Canon algorithm: C = A×B

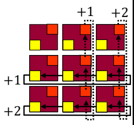
Product of dense matrixes of $\sqrt{N} \times \sqrt{N}$ elements
on a 2D torus of $\sqrt{P} \times \sqrt{P}$ processes

1 : Matrix partitioning: block I,J
on process I,J



2 : Pre-shifting of the initial blocks:

- A row I ($\in [0; \sqrt{P}-1]$) shifted of I step to the left ←
- B column J ($\in [0; \sqrt{P}-1]$) shifted of J step to the top ↑



3 : $\sqrt{P}$ steps: { local product $A_{IK} \times BK_J$ (partial sum of C_{IJ}) ;
1 notch shifting of A & B blocks: A_{IK} ←, B_{KJ} ↑ }

4 : Post-shifting of A and B blocks to restore input matrix partitioning

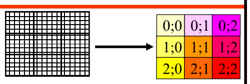
Dense matrix product on a 2D torus

Parallelisation scheme (C = A×B)

Canon algorithm: C = A×B

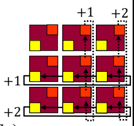
Product of dense matrixes of $\sqrt{N} \times \sqrt{N}$ elements
on a 2D torus of $\sqrt{P} \times \sqrt{P}$ processes

1 : Matrix partitioning: block I,J
on process I,J



2 : Pre-shifting of the initial blocks:

- A row I ($\in [0; \sqrt{P}-1]$) shifted of I notchs to the left ←
- B column J ($\in [0; \sqrt{P}-1]$) shifted of J notchs to the top ↑



3 : $(\sqrt{P} - 1)$ steps: { local product $A_{IK} \times BK_J$ (partial sum of C_{IJ}) ;
one notch shifting of A & B blocks: A_{IK} ←, B_{KJ} ↑ }

4 : 1 step: { local product $A_{IK} \times BK_J$ (partial sum of C_{IJ}) ;
post-shifting of A and B blocks to restore input matrix partitioning}

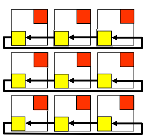
Dense matrix product on a 2D torus

Performance model (C = A×B)

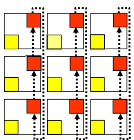
1 – Implementation with blocking communications (no overlapping)

Hypothesis:

1 - All computations are load balanced
2 - All computing units are identical
3 - All communications of one matrix shift are achieved in parallel
(only one send and one recv per process: sustainable by the switch)



and then:



No more communications at a time than with a ring of processes

→ Sustainable by the switch

4 - All communications cross the same « network »
(depends on the deployment !)

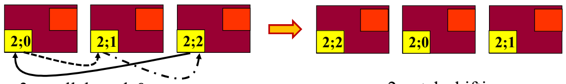
Dense matrix product on a 2D torus

Performance model (C = A×B)

1 – Implementation with blocking communications (no overlapping)

Pre- and post-shifting implementation on a cluster:

Each block can be moved with only one send and one recv
(no need to concatenate several elementary shifts on a cluster network)



3 parallel send-&-recv ≡ 2 notch shifting

⇨ $t_{pre-shifting-A} = t_s + (\frac{\sqrt{N}}{\sqrt{P}} \times \frac{\sqrt{N}}{\sqrt{P}}).t_w \approx \frac{N}{P}.t_w$

Considering blocking communications:

⇨ $\boldsymbol{t_{pre-shifting-A-B} \approx 2.\frac{N}{P}.t_w}$ **Same result for post-shifting**

Dense matrix product on a 2D torus

Performance model (C = A×B)

1 – Implementation with blocking communications (no overlapping)

- Pre-shifting (A & B): $2.\frac{N}{P}t_w$
- 1 step computations: $\frac{\sqrt{N}}{\sqrt{P}}.\frac{\sqrt{N}}{\sqrt{P}}.(2.\frac{\sqrt{N}}{\sqrt{P}}.t_{flop}) = 2.\left(\frac{N}{P}\right)^{\frac{3}{2}}.t_{flop}$
  (P processes in parallel)
- 1 step circulation: $2.(\frac{\sqrt{N}}{\sqrt{P}}.\frac{\sqrt{N}}{\sqrt{P}}).t_w = 2.\frac{N}{P}t_w$
  (blocking communications)
- Post-shifting (A & B): $2.\frac{N}{P}t_w$

Complete distributed computation:

$$T_{C\ comput}^{distributed} \approx t_{pre-shifting} + (\sqrt{P}\text{-}1).(t_{1-step-comput} + t_{1-step-circulation}) + (t_{1-step-comput} + t_{post-shifting})$$

Dense matrix product on a 2D torus

Performance model (C = A×B)

1 – Implementation with blocking communications (no overlapping)

- Pre-shifting (A & B): $2.\frac{N}{P}t_w$
- 1 step computations: $\frac{\sqrt{N}}{\sqrt{P}}.\frac{\sqrt{N}}{\sqrt{P}}.(2.\frac{\sqrt{N}}{\sqrt{P}}.t_{flop}) = 2.\left(\frac{N}{P}\right)^{\frac{3}{2}}.t_{flop}$
  (P processes in parallel)
- 1 step circulation: $2.(\frac{\sqrt{N}}{\sqrt{P}}.\frac{\sqrt{N}}{\sqrt{P}}).t_w = 2.\frac{N}{P}t_w$
  (blocking communications)
- Post-shifting (A & B): $2.\frac{N}{P}t_w$

Complete distributed computation:

$$T_{C\ comput}^{distributed} \approx 2.\frac{N}{P}t_w + (\sqrt{P}\text{-}1).(2.\left(\frac{N}{P}\right)^{\frac{3}{2}}.t_{flop} + 2.\frac{N}{P}t_w) + (2.\left(\frac{N}{P}\right)^{\frac{3}{2}}.t_{flop} + 2.\frac{N}{P}t_w)$$

Performance model (C = A×B)

1 – Implementation with blocking communications (no overlapping)

- Pre-shifting (A & B): $2.\frac{N}{P}t_w$
- 1 step computations: $\frac{\sqrt{N}}{\sqrt{P}}.\frac{\sqrt{N}}{\sqrt{P}}.(2.\frac{\sqrt{N}}{\sqrt{P}}.t_{flop}) = 2.\left(\frac{N}{P}\right)^{\frac{3}{2}}.t_{flop}$
  (P processes in parallel)
- 1 step circulation: $2.(\frac{\sqrt{N}}{\sqrt{P}}.\frac{\sqrt{N}}{\sqrt{P}}).t_w = 2.\frac{N}{P}t_w$
  (blocking communications)
- Post-shifting (A & B): $2.\frac{N}{P}t_w$

Complete distributed computation:

$$T_{C\ comput}^{distributed} \approx 2.\frac{N^{3/2}}{P}t_{flop} + 2.(\sqrt{P}+1).\frac{N}{P}t_w$$ **with: $P > 1$**

Performance model (C = A×B)

1 – Implementation with blocking communications (no overlapping)

Complete distributed computation:

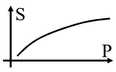
$$T_{C\ comput}^{distributed} \approx 2.\frac{N^{3/2}}{P}t_{flop} + 2.(\sqrt{P}+1).\frac{N}{P}t_w$$

Speedup:

$$S(N,P) = \frac{T_{C\ comput}^{Seq}}{T_{C\ comput}^{distributed}} \approx \frac{2.N^{3/2}.t_{flop}}{2.\frac{N^{3/2}}{P}t_{flop} + 2.(\sqrt{P}+1).\frac{N}{P}t_w}$$

$$S(N,P) \approx P.\frac{1}{1+\frac{\sqrt{P}+1}{\sqrt{N}} \times \frac{t_w}{t_{flop}}}$$

With $N = N_0$



Again: $\lim_{N\to\infty} S(N,P_0) = P_0$

When pb size increase, speedup increases and tends to ideal speedup

Performance model (C = A×B)

2 – Implementation with non-blocking communications

If we assume the switch can achieve A and B communications in parallel:

- Pre-shifting (A & B): $2.\frac{N}{P}t_w$ → $\frac{N}{P}t_w$
- 1 step computations: $2.\left(\frac{N}{P}\right)^{\frac{3}{2}}.t_{flop}$ (unchanged)
  (P processes in parallel)
- 1 step circulation: $2.\frac{N}{P}t_w$ → $\frac{N}{P}t_w$
  (blocking communications)
- Post-shifting (A & B): $2.\frac{N}{P}t_w$ → $\frac{N}{P}t_w$

Complete distributed computation:

$$T_{C\ comput}^{distributed} \approx 2.\frac{N^{3/2}}{P}t_{flop} + 2.(\sqrt{P}+1).\frac{N}{P}t_w$$

$$\rightarrow 2.\frac{N^{3/2}}{P}t_{flop} + (\sqrt{P}+1).\frac{N}{P}t_w$$

Dense matrix product on a 2D torus

Performance model (C = A×B)

2 – Implementation with non-blocking communications

Complete distributed computation:

$$T_{C\ comput}^{distributed} \approx 2.\frac{N^{3/2}}{P} t_{flop} + (\sqrt{P}+1).\frac{N}{P} t_w$$

Speedup:

$$S(N,P) = \frac{T_{C\ comput}^{Seq}}{T_{C\ comput}^{distributed}} \approx \frac{2.N^{3/2}.t_{flop}}{2.\frac{N^{3/2}}{P} t_{flop} + (\sqrt{P}+1).\frac{N}{P} t_w}$$

$$\boldsymbol{S(N,P) \approx P.\frac{1}{1+\frac{\sqrt{P}+1}{2.\sqrt{N}} \times \frac{t_w}{t_{flop}}}}$$

Speedup increases faster

Again: $\lim_{N\to\infty} S(N,P_0) = P_0$

When pb size increase, speedup increases and tends to ideal speedup

Dense matrix product on a 2D torus

Performance model (C = A×B)

Comparison 1D Ring / 2D Torus solutions:

	1D ring	**2D torus**
Blocking comm	$T_{par}(P) \approx 2.\frac{N^{3/2}}{P}.t_{flop} + N.t_w$	$T_{par}(P) \approx 2.\frac{N^{3/2}}{P}.t_{flop} + 2.(\sqrt{P}+1)\frac{N}{P}.t_w$
Non-blocking: A & B circ. in parallel		$T_{par}(P) \approx 2.\frac{N^{3/2}}{P}.t_{flop} + (\sqrt{P}+1)\frac{N}{P}.t_w$

2D torus solution *should be* faster when

- P ≥ 3 *with blocking comm* → P ≥ 2^2
- P ≥ 2 *with blocking comm* → P ≥ 2^2

Il the assumptions on the network switch are right…

Distributed algorithms on multi-dimensional topologies

Dense matrix product on a 1D ring
Dense matrix product on a 2D torus
Hyper-quicksort on a kD hypercube

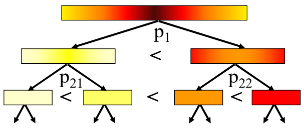
- **Parallelisation scheme**

Hyper-quicksort on a kD hypercube

Principe séquentiel

Principe :

- Tri récursif
- Diviser-pour-régner
- Séparation des données selon des valeurs *pivots*



Performances :

pire cas : $\left(\frac{N.(N-1)}{2}\right).t_{comp} \rightarrow O(N^2)$

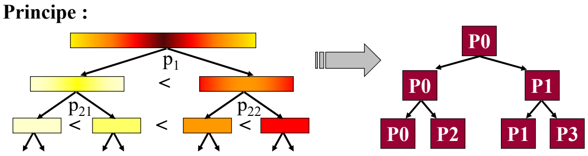
moyen cas (Knuth) : $\rightarrow O(2.N.\log(N))$

meilleur cas : $\left(N.\log_2(N) - 2.N + 1\right)t_{comp} \rightarrow O(N.\log(N))$

Hyper-quicksort on a kD hypercube

Parallélisation naïve

Principe :



On crée un nouveau processus et on utilise un nouveau processeur à chaque appel récursif (en réutilisant les anciens).

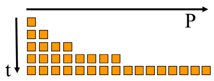
Simple mais inefficace !

Hyper-quicksort on a kD hypercube

Parallélisation naïve

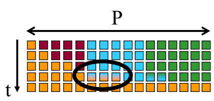
1 - Faiblesse conceptuelle :

Algorithme pas pleinement parallèle
→ Trouver plus parallèle !

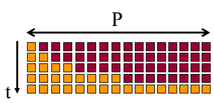


2 - Faiblesse de mise en œuvre :

Ressources (CPU) partagées :
→ Temps de création dynamique et de re-répartition de processus

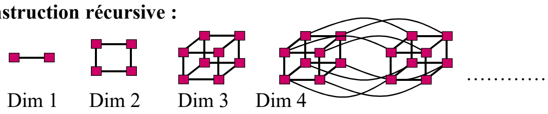


Ressources (CPU) allouées au lancement de l'application :
→ Gaspillage !

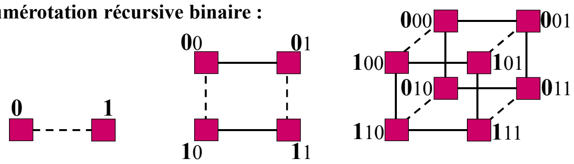


Rappel des propriétés des Hyper-Cubes

Construction récursive :



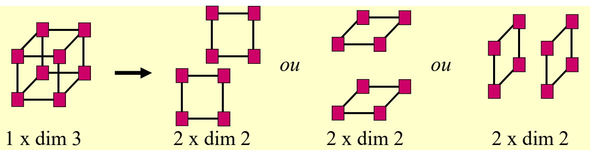
Numérotation récursive binaire :



Rappel des propriétés des Hyper-Cubes

Décomposition en sous-hypercube :

1 hypercube de dimension n coupé par un hyper-plan donne 2 sous-hypercubes de dimension n-1 :

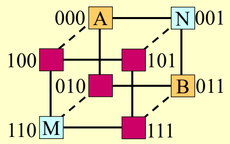


→ Les algorithmes de routages restent les mêmes dans toutes les parties de l'hyper-cube

Rappel des propriétés des Hyper-Cubes

Distance entre nœuds :

Le nombre de bits différents dans les adresses de deux nœuds donne leur distance (si la numérotation a suivi la construction récursive)



Ex : A : 000, B : 011
d(M,N) = 2

Ex : M : 110, N : 001
d(M,N) = 3

→ Les algorithmes de routages seront à base de calculs simples (et rapides)

Hyper-quicksort on a kD hypercube

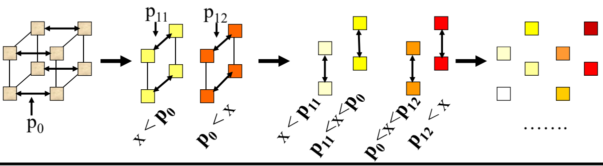
Principe de parallélisation sur Hyper-Cube

Objectif :

- **Tous les processeurs doivent travailler tout le temps !**
- S'inspirer d'un tri séquentiel rapide (*O(N.log(N))*) : quick-sort

→ Trouver un schéma de parallélisation avec comm. optimisées (peu de comm, ou comm locales)

Idée : Quick-sort : algo récursif ↔ topologie récursive : Hyper-cube



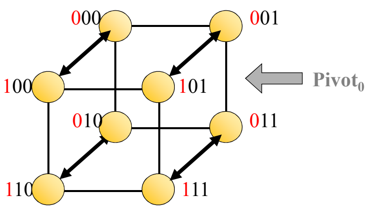
Hyper-quicksort on a kD hypercube

Principe de parallélisation sur Hyper-Cube

Init : Chaque nœud charge N/P données en local

Etape 0 :

- Choix de 2^0 pivots pour les 2^0 hypercubes de dimension d-0
- Séparation selon le pivot sur chaque nœud de l'hypercube
- Echange des listes inférieures et supérieures en dimension d-0
- Fusions locales des listes conservées et des listes reçues

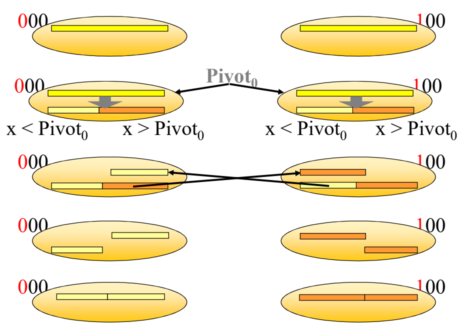


Communication avec voisin en dimension d-0 : seul le bit d-0 diffère

Hyper-quicksort on a kD hypercube

Principe de parallélisation sur Hyper-Cube

Détails de l'étape 0 sur PE-000 et PE-100 :



Séparations locales

Echanges de sous-listes

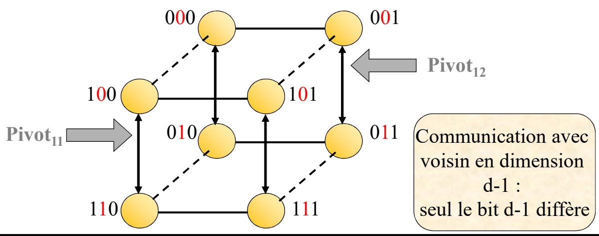
Fusion des sous-listes conservées et reçues

Hyper-quicksort on a kD hypercube

Principe de parallélisation sur Hyper-Cube

Etape 1 :

- Choix de 2^1 pivots pour les 2^1 hypercubes de dimension d-1
- Séparation selon un pivot sur chaque nœud des 2^1 hypercubes
- Echange des listes inférieures et supérieures en dimension d-1
- Fusions locales des listes conservées et des listes reçues

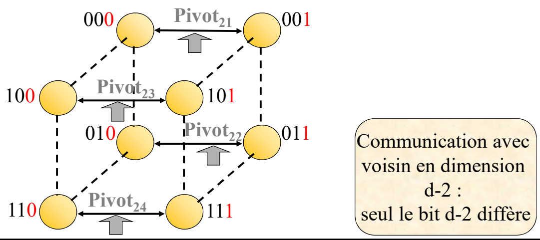


Hyper-quicksort on a kD hypercube

Principe de parallélisation sur Hyper-Cube

Etape 2 :

- Choix de 2^2 pivots pour les 2^2 hypercubes de dimension d-2
- Séparation selon un pivot sur chaque nœud des 2^2 hypercubes
- Echange des listes inférieures et supérieures en dimension d-2
- Fusions locales des listes conservées et des listes reçues



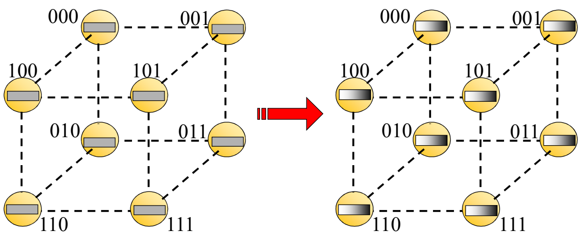
Hyper-quicksort on a kD hypercube

Principe de parallélisation sur Hyper-Cube

Etape finale :

Chaque processeur tri en local sa liste finale

→ ex : quick-sort local et séquentiel sur chaque processeur



Hyper-quicksort on a kD hypercube

1ère implantation sur Hyper-Cube

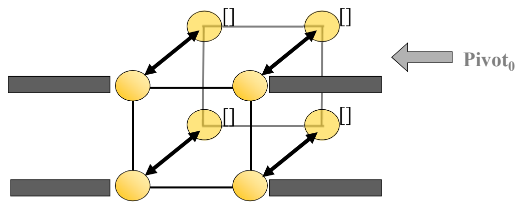
```
HcubeQuickSort(double *A, int d)
{
 int i;                        /* Compteur de dimension. */
 double *Ainf, *Asup, *Abuf;   /* Tables de données.     */
 double x;                     /* Pivot.                 */
 for (i = d-1; i >= 0; i--) {  /* Pour chaque dimension: */
   x = choix_pivot(me,i);      /* - Partitionnement local*/
   partitioner(A,x,Ainf,Asup);
   if (me & exp2(i) == 0) {    /* - Communications       */
     asend(Asup,me | exp2(i));
     recv(Abuf,me | exp2(i));
     union(Ainf,Abuf,&A);
   } else {
     asend(Ainf,me & ~exp2(i));
     recv(Abuf,me & ~exp2(i));
     union(Abuf,Asup,&A);
   }
 }
 QuickSortSequentiel(A);       /* Tri final des donnes   */
}                              /* locales                */
```

Hyper-quicksort on a kD hypercube

1ère implantation sur Hyper-Cube

Faiblesse de ce premier hyper-quicksort :

Si *mauvais choix* de pivot : déséquilibre définitif !
→ processeurs surchargés et processeurs à vide

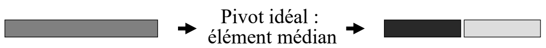


→ **Soigner le choix du pivot.**

Hyper-quicksort on a kD hypercube

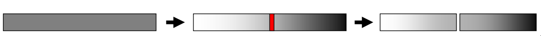
Parallélisation optimisée sur H-Cube

Choix de pivot optimisé :



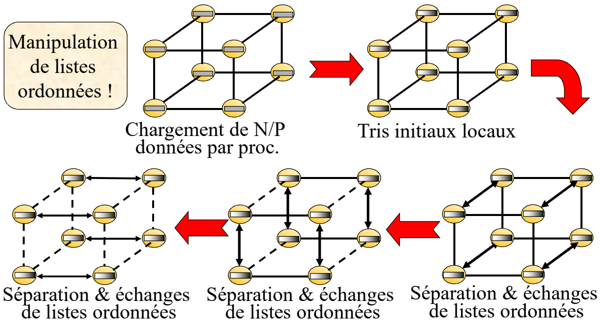
En séquentiel : on approxime l'élément médian, ex :
- l'élément médian des 5 premiers,
- l'élément médian d'un échantillonnage, ….

En parallèle :
- on trie initialement les éléments locaux
  → on prend l'élément médian ! LocalTab[N/P/2]
- on suppose une distribution homogène sur tous les PEs
  → le pivot idéal d'un PE est un très bon pivot pour tous !



Parallélisation optimisée sur H-Cube

Principe de l'hyper-quicksort optimisé :



$2^{ième}$ implantation sur Hyper-Cube

Implantation de l'hyper-quicksort optimisé :

```
HcubeQuickSort(double *A, int d)
{
 int i;                          /* Compteur de dimension. */
 double *Ainf, *Asup, *Abuf;     /* Tables de données.     */
 double x;                       /* Pivot.                 */
 QuickSortSequentiel(A);         /* Tri initial des donnes */
                                 /* locales.               */
 for (i = d-1; i >= 0; i--) {    /* Pour chaque dimension: */
   x = choix_pivot(me,i);        /* - Partitionnement local*/
   partitioner(A,x,Ainf,Asup);
   if (me & exp2(i) == 0) {      /* - Communications       */
     asend(Asup,me | exp2(i));
     recv(Abuf,me | exp2(i));
     union_ordonne(Ainf,Abuf,&A);
   } else {
     asend(Ainf,me & ~exp2(i));
     recv(Abuf,me & ~exp2(i));
     union_ordonne(Abuf,Asup,&A);
   }
 }
}
```

Distributed algorithms on
multi-dimensional topologies

End