European Option Pricing on a GPU Cluster
(ANR project « GCPMF »)

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1 – Objectives
Objectives

Main objectives and difficulties

Final objective:

High speed European contract pricing (for hedging)
- using Monte-Carlo computations,
- using a clusters of multi-cores (GPUs or multi-core CPUs)

Scientific and technical locks:
- Design a parallel algorithm for an European option pricer on GPU and cluster of GPU,
- Design and implement a right and efficient parallel RNG,
- Find out the right compromise between speedup, size up, result accuracy and energy consumption
2 – RNG parallelization and comparison
RNG parallelization and comparison

Principles

• RNG on CPU vs RNG on GPU
  1. Data transfer
  2. Faster if Parallel

• Parallel RNGs: two efficient alternatives for GPU
  1. Period Splitting
  2. Parametrization
Parallel RNG: PLCG

• Parallelization with parametrization

\[ X_n = a.X_{n-1} \% m \]

• « a » is the parameter

• Chose an « a » for each sub-stream such that:
  1. Good parallel independence inter-streams
     Michael Mascagni SPRNG
  2. Good sequential independence intra-stream
     Donald E. Knuth Art of Computer Programming
  3. Total period: \( 2^{41} \) (int 32) and \( 2^{65} \) (float 64)
RNG parallelization and comparison

Parallel RNG: CMRG

- Pierre L’Ecuyer: Combination of two MRGs (total period: $2^{185}$)

\[
X_n = a_1 X_{n-1} + a_2 X_{n-2} + a_3 X_{n-3} \pmod{m} \\
X'_n = a'_1 X'_{n-1} + a'_2 X'_{n-2} + a'_3 X'_{n-3} \pmod{m'}
\]

\[
A = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
a_3 & a_2 & a_1
\end{pmatrix} \quad A' = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
a'_3 & a'_2 & a'_1
\end{pmatrix}
\]

For example, if we want:

- one RNG/trajectory,
- $2^{18}$ trajectories/GPU,
- maximum of 16 = $2^4$ GPUs.

\[
IP = 2^{185}/2^{22} = 2^{163} \text{ and } X_\_1 = (X_1, X_2, X_3)^T, X'_\_1 = (X'_1, X'_2, X'_3)^T
\]

\[
X_i = (A^{IP})_i.X \pmod{m} \quad X'_i = (A'^{IP})_i.X' \pmod{m'}
\]
RNG parallelization and comparison

Parallel RNG: Results

• Results Accuracy

<table>
<thead>
<tr>
<th>Strike « K »</th>
<th>Real</th>
<th>PLCG</th>
<th>PLCG error</th>
<th>CMRG</th>
<th>CMRG error</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.0610</td>
<td>0.0621</td>
<td>0.0023</td>
<td>0.0611</td>
<td>0.0024</td>
</tr>
<tr>
<td>90</td>
<td>0.5068</td>
<td>0.5115</td>
<td>0.0078</td>
<td>0.5088</td>
<td>0.0008</td>
</tr>
<tr>
<td>100</td>
<td>2.1723</td>
<td>2.1862</td>
<td>0.0178</td>
<td>2.1740</td>
<td>0.0177</td>
</tr>
<tr>
<td>110</td>
<td>5.9208</td>
<td>5.9433</td>
<td>0.0299</td>
<td>5.9159</td>
<td>0.0296</td>
</tr>
<tr>
<td>120</td>
<td>11.8810</td>
<td>11.9023</td>
<td>0.0407</td>
<td>11.8775</td>
<td>0.0406</td>
</tr>
</tbody>
</table>

• Speedup Results for PRNG

<table>
<thead>
<tr>
<th>Machine</th>
<th>Gaussian sample for PLCG generation/second</th>
<th>Gaussian sample for CMRG generation/second</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU</td>
<td>2.24 Kg/s</td>
<td>2.03 Kg/s</td>
</tr>
<tr>
<td>GPU</td>
<td>883.58 Kg/s</td>
<td>239.80 Kg/s</td>
</tr>
</tbody>
</table>
3 – Parallel algorithm and implementations
Parallel algorithm and implementations

Parallel algorithm (1)

Parallel programming paradigms:
- Coarse grain: message passing (MPI on PC cluster)
- Medium grain: CPU-multithreading (OpenMP on multi-cores)
- Fine grain: GPU-multithreading (CUDA on GPU)

Strategy:
- Avoid concurrent input file accesses
- Minimize data transfer between CPU and GPU memories.
- Common algorithm for multi-core-CPU and GPU clusters.
- When limited by the GPU memory: “size up + speedup”

![Graph showing performance metrics](image)
Parallel algorithm and implementations

Parallel algorithm (2)

1 - Input data reading on $P_0$
2 - Input data broadcast from $P_0$
3 - Parallel and independent RNG initialization
4 - Parallel and independent Monte-Carlo computations
5 - Parallel and independent partial results computation
6 - Partial results reduction on $P_0$ and final price computation
7 – Print results and perfs
Parallel algorithm and implementations

GPU/CPU code comparison (1)

OpenMP parallelization on multi-core CPU: split the external loop

```c
void ActStock(double sqrtldt)
{
    int StkIdx, yIdx, xIdx; // Loop indexes
    #pragma omp parallel private(StkIdx,yIdx,xIdx)
    {
        for (StkIdx = 0; StkIdx < NbStocks; StkIdx++) {
            Parameters_t *parPt = &par[StkIdx];
            // Process each trajectory
            #pragma omp for
            for (yIdx = 0; yIdx < Ny; yIdx++)
                for (xIdx = 0; xIdx < Nx; xIdx++) {
                    float call;
                    // - First pass
                    call = ......;
                    // - The passes that remain
                    for (int stock = 1; stock <= StkIdx ; stock++)
                        call = ......;
                    // Copy result in the global GPU memory
                    TabStockCPU[StkIdx][yIdx][xIdx] = call;
                }
        }
    }
}
float TabStockCPU[NbStocks][Ny][Nx]```
Parallel algorithm and implementations

GPU/CPU code comparison (2)

CUDA parallelization on GPU: one kernel work on one trajectory

```c
__global__ void Actual_kernel(void)
{
    float call, callBis;

    // Computes the indexes and copy data into multipro sh. memory
    int xIdx = threadIdx.x + blockIdx.x*BlockSizeX;
    int yIdx = blockIdx.y;
    __shared__ float InputLine[Nx];
    __shared__ float BrownLine[Nx];
    InputLine[xIdx] = TabStockInputGPU[StkIdx][yIdx][xIdx];
    GaussLine[xIdx] = TabGaussGPU[0][yIdx][xIdx];

    // First pass
    call = ......;
    callBis = call;

    // The passes that remain
    for (int stock = 1; stock <= StkIdx; stock++) {
        GaussLine[xIdx] = TabGaussGPU[stock][yIdx][xIdx];
        call = callBis*......;
        callBis = call;
    }

    // Copy result in the global GPU memory
    TabStockOutputGPU[StkIdx][yIdx][xIdx] = call;
}
```

float TabStockOutputGPU[NbStocks][Ny][Nx]
CUDA parallelization on GPU: one kernel work on one trajectory

```c
void ActStock(double sqrtdt)
{
    // GPU thread management variables
    dim3 Dg, Db;

    // Set thread Grid and blocks features
    Dg.x = Nx/BlockSizeX; Dg.y = Ny; Dg.z = 1;
    Db.x = BlockSizeX; Db.y = 1; Db.z = 1;

    // Transfer a float version of the time increment on the GPU
    float sqrtdtCPU = (float) sqrtdt;
    cudaMemcpyToSymbol(sqrtdtGPU,&sqrtdtCPU,sizeof(float),0,
                        cudaMemcpyHostToDevice);

    // For each stock: transfer its index on the GPU and compute
    // its actualization (process all trajectories)
    for (int s = 0; s < NbStocks; s++) {
        cudaMemcpyToSymbol(StkIdx,&s,sizeof(int),0,
                            cudaMemcpyHostToDevice);
        Actual_kernel<<<Dg,Db>>>(()); // Run the GPU computation
    }
}
```

→ Identical data structures for CPU and GPU versions when using one GPU-thread per trajectory
Parallel algorithm and implementations

Compilation

OpenMPI + Cuda:

```
nvcc --host-compilation C++
   -O3 -I/opt/openmpi/include
   -DOMPI_SKIP_MPICXX -c X.cu

nvcc -O3 -L/opt/openmpi/lib
   -o pricer X.o Y.o .... -lmpi -lm
```

OpenMPI + OpenMP

```
g++ -O3 -fopenmp -I/opt/openmpi/include
   -c X.cc

g++ -O3 -fopenmp -L/opt/openmpi/lib
   -o pricer X.o Y.o ... -lmpi -lm
```

This is a basic C++ code.

Compilation with CUDA code is easy.
4 – Experiments and performance analysis
Experiments and performance analysis

Computing perf (1)

Pricing execution time with PLCG

Good scaling on both systems.
Experiments and performance analysis
Computing perf (2)

Pricing execution time with CMRG

Good scaling on both systems.
GPU time is impacted by the RNG.
Experiments and performance analysis

Computing perf (3)

Pricing speed with PLCG

Asian Pricer on clusters of GPU and CPU with PLCG

Computation speed is independent of the problem size.
Experiments and performance analysis

Computing perf (4)

Pricing speed with CMRG

Computation speed is independent of the problem size.
GPU computation speed is impacted by the RNG choice.
Experiments and performance analysis

Computing perf (5)

Pricing speedup with PLCG

With 16 GPU nodes: speedup vs 1-core-CPU reaches 1636, speedup vs 1-node-CPU reaches 707, speedup vs 256-nodes-CPU cluster reaches 2.83.
Experiments and performance analysis

Computing perf (6)

Pricing speedup with CMRG

With 16 GPU nodes: speedup vs 1-core-CPU reaches 1192, speedup vs 1-node-CPU reaches 515, speedup vs 256-nodes-CPU cluster reaches 2.09.
Experiments and performance analysis

Energetic perf (1)

Energy consumption with PLCG

Air conditioned has not been considered.
16-nodes GPU cluster consumes 28.3 times less than a 256-nodes CPU cluster.
Experiments and performance analysis

Energetic perf (2)

Energy consumption with CMRG

Air conditioned has not been considered.
16-nodes GPU cluster consumes 19.1 times less than a 256-nodes CPU cluster.
Experiments and performance analysis

Energetic perf (3)

Effectiveness of computing energy with PLCG

Currently, air conditioning has not been considered.
Effectiveness of computing energy is 28.3 times higher on the 16-nodes GPU cluster.
Experiments and performance analysis

Energetic perf (4)

Effectiveness of computing energy with CMRG

Currently, air conditioning has not been considered.
Effectiveness of computing energy is 19.0 times higher on the 16-nodes GPU cluster.
Experiments and performance analysis

Bi-core CPU cluster vs GPU cluster

Global comparison of GPU and CPU clusters

<table>
<thead>
<tr>
<th>Bi-core CPU cluster</th>
<th>GPU cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>256 bi-core Xeon 3075, RAM 4Go, cache 4Mo</td>
<td>16 bi-core Intel E8200, RAM 4Go, cache 6Mo, Asus GeForce 8800 GT</td>
</tr>
<tr>
<td>1 CISCO 256-ports switch, gigabit-eth</td>
<td>1 DELL 24-ports switch, gigabit-eth</td>
</tr>
</tbody>
</table>

Benchmark: 1024x1024 trajectories
40 stocks European contract pricing
RNG: PLCG

58.70s
464.3Wh

20.72s 
16.4Wh

Speedup = 2.83
Saving = 28.3

GPU cluster is 2.83x28.3 = 80.1 times more efficient
5 – Conclusion and perspectives
Conclusion & Perspectives

Developments:
- OpenMPI + CUDA + “C+” were compatible.
- Learning CUDA + all dev ≈ 3 weeks (2 people).
- Debug on GPU was hard.

Performances:
- Good scaling
- GPU cluster computes faster
- GPU cluster consumes less energy
- Too high quality RNG is harmful
- PLCG + 16-node GPU cluster
  - 1636 times faster than 1-core CPU
  - 2.8 times faster than 256-node CPU cluster
  - consumes 28.3 times less than 256-node CPU cluster

Next steps:
- Experiment others Monte-Carlo simulations
- Design algorithm with better perf & mixed perf (speedup x energy saving)
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Questions ?
Building a cluster of GPUs

Hardware choice

GPU on each node: ASUS GeForce 8800 GT

Product model: EN8800GT/G/HTDP/512M
- Multiprocessors: 14
- Stream processors: 112
- Core clock: 600 MHz
- Memory clock: 900 MHz
- Memory amount: 512 MB
- Memory interface: 256-bit
- Memory bandwidth: 57.6 GB/sec
- Texture fill rate: 33.6 billion/sec

CPU on each node: 1 processor dual-cores Intel E8200, 2.66 GHz
- Front side bus: 1333 MHz
- RAM: 4Go DDR3, cache: 6Mo

Asynchronous communications and Cuda 1.1 supported