AN EMPIRICAL EVALUATION FRAMEWORK FOR QUALIFYING DYNAMIC NEURAL FIELDS

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ABSTRACT
In this paper, the behavior of dynamic neural fields is studied through the lens of performance. As an alternative to the currently available analytical instruments, an empirical evaluation methodology is proposed in order to examine the dynamic quality of a field. This consists of simulating the field through various key scenarios and compare the observed behavior to an optimal expected one. Some desired effects concerning the evolution of an ideal field are inspected, and a performance criterion is defined accordingly. Practically, this approach implements a generic benchmark framework for qualifying neural fields, allowing to inspect the evolution of the model in different key situations. The presented methodology provides a basis for a methodological evaluation of the computational power of neural fields, when they serve as a basis of decision processes. In a such integrated system, our approach allows to tune the free parameters of the field equation according to the behavior expected from them.

KEY WORDS
dynamic neural fields, empirical evaluation

1 Introduction

Dynamic neural fields are a mathematical formalism describing the activity of a population of neurons. More specifically, it consists of modelling the temporal evolution of the membrane potential, $u(x,t)$, for each neural unit x of a set of units, at any moment of time t. The field is supposed to respond to the distribution of some other external activity, $i(x,t)$. This response can be interpreted in a classical perspective as a pattern recognition task performed by the unit x seeking to extract relevant information from a given input i. Globally, the field response aims to choose the best responding units of the population. The $u(x,t)$ can be therefore viewed as a *contrasted* version of $i(x,t)$, enhancing the neural activity at relevant places in the field. The so-called *lateral* connections within the field, with typical on-center/off-surround shape, involved in forming the response $u(x,t)$, can be then interpreted as a locally driven competition process. In [1] for example, this paradigm leads to equation 1.

The dynamics of such neural field mechanisms have been studied from a mathematical point of view [1, 2] and results concerning the ability to form patches of u activities from the i distribution have been obtained for constant i over the field. Our goal is to investigate the fields beyond this limit, through empirical measurements. The usages of neural fields are manifold. For example, they are involved in biological modeling, where time delays between several fields have been addressed [3]. The analysis of the model dynamics is performed in order to be compared quantitatively with in vivo recordings. Neural fields have also been used for controlling a robotic arm [4, 5], for a multimodal integration of sensory-motor control [6] and for visual attention [7].

$$\tau \frac{\mathrm{d}u(x,t)}{\mathrm{d}t} = -u(x,t) + \int_{x'} w(|x-x'|) f(u(x',t)) \mathrm{d}x' + i(x,t) + h \quad (1)$$

Using theoretical models in real application for describing the dynamics of complex systems requires a good understanding of their intrinsic properties. Studying them in detail is usually carried through two different approaches: one analytical, using mathematical formalisms, and another empirical, based on experimental observations of the model. In the latter case, the behavior of the system is acquired by simulating it in various conditions. The recorded data is then analysed in a quantitative manner in order to express qualitative statements. This paper is adopting this second approach to analyse neural field properties.

Mathematical research studies are inclined towards examining the behavior of the neural fields with uniform input over space. Even if this particular case may seem theoretically inspiring [8], *input varying both in space and time is definitely the dominant case in most applicative frameworks simulations*.

Considering the similarity between the real data and the simulated one as a performance criteria of a biologically inspired model may prove often too restrictive, either because it is rather difficult to acquire the necessary biological data or because the exact reproduction of it is not aimed by the model. In this case, performance criteria of such models need to be asserted by an external knowledge. However, experimental data (both biological and simulated) gives us an insight about the optimal states of the model. As a gen-

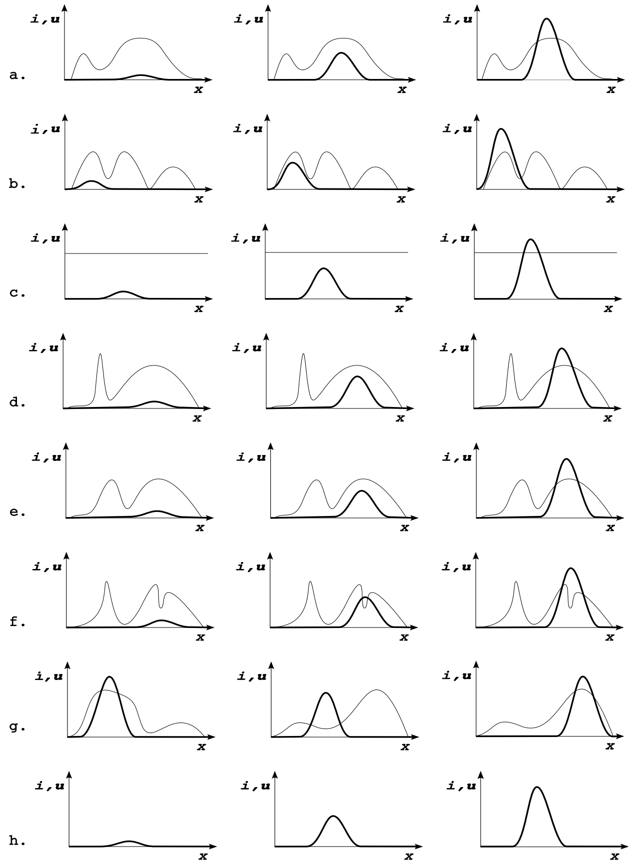

Figure 1. Some expected effects of a qualitative one-dimensional neural field activity (the thick plot), in response to various inputs (the thin plot). For each row, the temporal evolution of the field is shown from left to right. The field is assumed to generate a single bump of activity. a. selectivity, b. competition, c. competition (plateau effect), d. competition (strong detractor), e. competition (weak detractor), f. collaboration, g. adaptation, h. decision. (extended from [9])

eral remark, neural fields respond to a given input, i, by increasing their neural activity, u, where i is "locally strong".

Figure 1 presents the effects deired when the field is used as a computational module. This functional perspective, also supposed in [9], is not addressed by mathematical analysis, as far as we know.

Let us highlight the case of null input scenario (h). The field should not cease activating in this particular case, since we consider it should detect at least one region of information from the input. In other words, the fact that the input is absent should not prevent the field from focusing activity somewhere. Rather than choosing among different options offered by the input, such a field decides to actually create information "out of nothing" in order to explore the environment, and not only to exploit it. Besides, we have already experimented this as a crucial feature in online learning architectures [6], since it allows the learning to start from scratch. A specific numerical instrument to capture and measure all these effects along the evolution of a neural field is described in section 3. Nevertheless, any other measurement can be used in the experimental framework, depending on the actual purpose of the neural field designer. The methodology proposed by this generic approach is detailed in the next section.

2 Method

Studying the dynamics of neural fields may first require an analytical resolution of the equations describing the field. One could then check whether the solution attains an assumed optimal criterion. As an alternative to this perspective, the following empirical approach is proposed to address qualitative aspects of the fields. Briefly described, this consists in examining the behavior of a particular field through simulation and evaluate at each step the distance between the current state of the field and a reference one considered as optimal. A specific optimality criterion is defined and analysed in detail in the next section. Finally, what interests the most is whether stabilized states of the field succeed in being also qualitative ones.

Practically, such a methodology is implemented as a generic benchmark framework. A battery of predefined inputs composes the scenarios pool, while the above mentioned measuring tools account for the quantification instruments pool. This allows to quickly describe any benchmarking experiment simply by defining a scenario and a desired measuring instrument. Running multiple experiments would then give an empirical evaluation of the field average behavior in certain key situations.

Such empirical estimations provide a powerful assistance for a designer who needs to inspect a neural field property for a specific purpose, as robotics or bio-inspired computational modelling. In this case, parameter tuning (e.g. lateral excitatory and inhibitory weight kernel sizes, relative strengths of input and lateral influence etc.) is critical and is often actually set up after successive trials.

3 A quantification instrument for neural fields

For each simulation step of an experiment, the field activity is quantified relative to a set of expected properties. Rather than a boolean expression, the comparison between the current field distribution and an assumed optimal one should be a function able to gradually measure the convergence of one towards the other.

As already stated, we consider neural fields as detectors of relevant regions of information within a given input: they grow so-called *bumps* of neural activity where the input is high. More formally, the following general property can be asserted:

(P): *Neural fields enhance their activity, u, where the input, i, is locally strong, ensuring that the distribution u evolves to a non-empty set of neural bumps.*

A quick review of the properties described in figure 1 would easily come to the conclusion that they are all the expression of (P). We consider thus the use of (P) as an implicit description of an optimality criterion both intuitive and practical, (P) being empirically perceived as a dominant behavior of an ideal field. Quantifying (P) would answer then *how well* this property is satisfied. This amounts to measure the capacity of the field to place neural bumps in the corresponding positions of locally high input activity.

Reexamining the formulation of the (P) property, one could notice two particular features implied by it. First, stating that "u is enhanced where i is locally strong" infers that the output is locally fitting the input. Second, the neural activity is supposed to "enhance" and maintain high in order to detect relevant information from the input. Practically, this enhancement translates into generating uniform bumps of high amplitude within the field. In summary, the field is expected to stabilize to distributions of u *well-fitted* to the input (WFT condition) and having a *well-formed* shape of sparsely distributed bumps (WFM condition). We assume that the neural bumps in the field are neither too close, nor too far one from the others (sparsity constraint).

Let $\mathcal{B}$ define the family of all well-formed neural activity distributions of bumps, whatever their shapes may be. Therefore, $\mathcal{B}$ would comprise of all the possible ways of positioning the bumps across the field, respecting the sparsity constraint. Depending on the definition of the optimal states, the bumps may have indeed specific shapes. For example, $\mathcal{B}$ could be defined as the field of all the bell-shaped curves that do not overlay within a fixed size neighborhood. However, the general analysis that follows these remarks is not constrained by any bump shape.

The following classical L^2 distance is further used to express the similarity between two distributions (e.g. the field input and the neural activity):

$$d(i,u) = \sqrt{\int_x ||i(x) - u(x)||^2 dx}$$

By abuse of notation, the distance between one distribution i and a set of distributions, $\mathcal{B}$, is defined as follows:

$$d(i,\mathcal{B}) = \min_{b \in \mathcal{B}} \{d(i,b)\}$$

With the aid of these notations, the set of best-fitted elements of $\mathcal{B}$ to a given input i can be expressed as:

$$\mathcal{I} = \{u^* \in \mathcal{B} \,|\, d(i,u^*) = d(i,\mathcal{B})\}$$

The two conditions emerging from (P) are evaluated. The interest of the method is to measure how well (P) is satisfied by the input-output field distributions. As a consequence of the above analysis, satisfying (P) within an imposed ε margin implies satisfying both (WFT) and (WFM) within the same bound.

A well-fitted distribution u should not only minimize its distance to the input i, but also be not too far from the best-fitted ones from $\mathcal{B}$, i.e. $\mathcal{I}$. In other words, u should

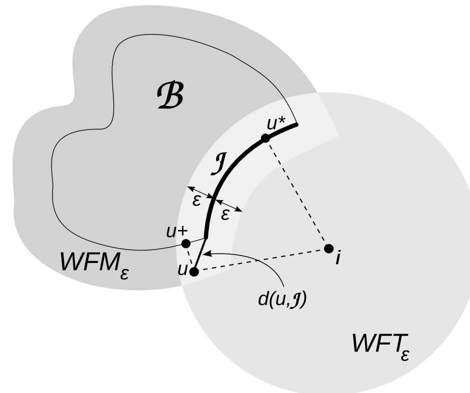

Figure 2. A geometrical interpretation of (P)

not fit i worse than any optimal distribution (WFT$_\varepsilon$) and it should also be close to a well-formed bump shape (WFM$_\varepsilon$):

$$d(i,u) \leq d(i,\mathcal{B}) + \varepsilon \qquad (\text{WFT}_\varepsilon)$$

$$d(u,\mathcal{B}) \leq \varepsilon \qquad (\text{WFM}_\epsilon)$$

Practically, in order to compute the distance from one particular distribution to the $\mathcal{B}$ set, an exhaustive research procedure is implemented to find the projection of i or u to $\mathcal{B}$. If, for example, $\mathcal{B}$ is defined (as it was used in our experiments) as the family set of bumps of a fixed bell-shape, scattered throughout the field in order not to overlay within a fixed size neighborhood, the procedure generates all the possible combinations of forming a valid distribution of bumps.

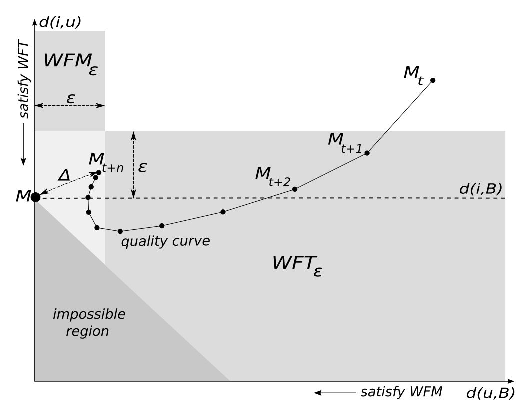

Figure 3. The plotting of (WFT) and (WFM) at different simulation steps gives the "quality curve" of the field.

Figure 3 provides the quality analysis picture of a generic experiment. The input i is kept constant until the state of the field stabilizes (e.g. for n steps in the figure) and at each simulation step t, the satisfaction of the (P) property is evaluated numerically. Let $M_t(d(i,u), d(u,\mathcal{B}))$ designate the numeric expression of the two intrinsic conditions of (P) at the corresponding step t, and $M(d(i,\mathcal{B}), 0)$ the ideal satisfaction of (P) by the field. A qualitative field behavior should imply a convergence of M_t towards M.

The method is also capable of tracking the two interrelated conditions of (P) during the relaxation of the field,

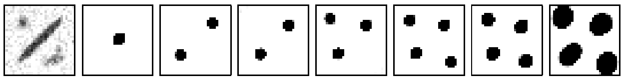

Figure 4. Influence of λ on a 2D neural fields dynamics. Input i (first image) is kept constant in all the experiments, and neural field evolution is evaluated. For each experiment, the stabilized distributions of the field, u, are shown from left to right, for different values of $\lambda(0, 0.1, 0.2, 0.4, 0.5, 0.6, 0.8, 1)$.

enabling a way to perceive distinctly their influence. In addition, given the fact that d is a distance in a functional space, the imposed conditions are sufficient to fully define a convergence criterion for (P). The following inequality thus stands from (WFT$_\varepsilon$) and (WFM$_\varepsilon$):

$$d(i, u) \geq d(i, \mathcal{B}) - \varepsilon$$

As a result of it, the impossible region in the figure 3 plot appears. Intuitively, it suggests that it is impossible for u to be very similar to i (thus very high-fitted) and also well-formed, and yet satisfy (P). This would hold only if i itself is close to $\mathcal{B}$. In particular, if i is an element of $\mathcal{B}$, the impossible region completely disappears.

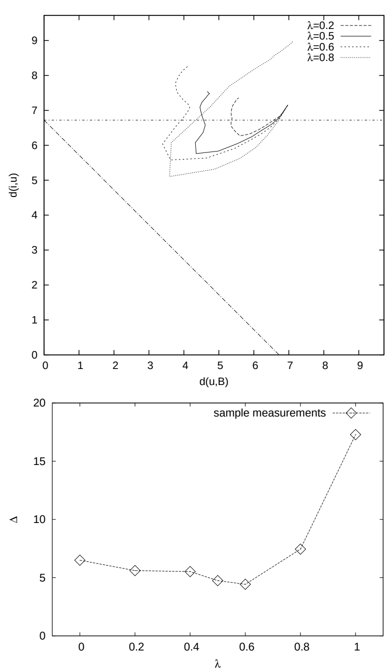

Figure 5. Quantification results. Above: quality curves (see figure 3) of neural fields evolutions for different λ values. Below: the distance Δ (see also figure 3) from the stabilized fields distributions to optimal ones.

4 Results

To illustrate the method's usage, we introduce a λ variable in the field equation to drive the smooth change of some of its parameters (not detailed here), in order to study the influence of this variable on the dynamic behavior of the field. Figure 4 presents a series of experiments to track this aspect, while figure 5 plots the corresponding quality curves of the field's evolution, for each simulated experiment. By imposing $\mathcal{B}$ (as mentioned in the previous section), thus by characterizing the optimal states, the method is capable to determine $\lambda^\star$ ($\lambda^\star = 0.6$ in figure 5), the locally optimal value of λ that best approaches the field to an ideal distribution satisfying (P).

5 Conclusion and future work

The quantification methodology proposed in this paper is a "passive" instrument for evaluating the evolution of neural fields in various particular cases. As suggested by the experimental results, an entirely new perspective can enable an active use of it for tuning the free parameters of the field in order to achieve a competitive behavior. As also a basic motivation for starting the current research, the next main goal will consider the use of this approach for finding qualitative neural fields in the benefit of a better design of cortically-inspired computational architectures.

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